

# Medium-induced flavor conversion and kaon spectra in electron-ion collisions

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Multiple scattering and induced parton splitting lead to a medium modification of the QCD evolution for jet fragmentation functions and the final hadron spectra. Medium-induced parton splittings not only lead to energy loss of leading partons and suppression of leading hadron spectra, but also modify the flavor composition of a jet due to induced flavor conversion via gluon emission, quark pair production and annihilation. Through a numerical study of the medium-modified QCD evolution, leading  $K^-$  strange meson spectra are found to be particularly sensitive to the induced flavor conversion in semi-inclusive deeply inelastic scatterings (SIDIS) off a large nucleus. The induced flavor conversion can lead to increased number of gluons and sea quarks in a jet and, as a consequence, enhance the leading  $K^-$  spectra to counter the effect of parton energy loss in SIDIS with large momentum fractions  $x_B$  where the struck quarks are mostly valence quarks of the nucleus.

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In the study of quark gluon plasma (QGP) produced in relativistic heavy-ion collisions, jet quenching has been established as one of the evidences for the formation of strongly coupled QGP in experiments at both the Relativistic Heavy-ion Collider (RHIC) and the Large Hadron Collider (LHC) [1, 2]. The phenomenon as manifested in the suppression of jets and final hadron spectra at large transverse momentum is a consequence of parton energy loss induced by multiple scatterings during the propagation of the initial hard partons through the dense medium of QGP [3–10]. The same phenomenon is also predicted to happen in semi-inclusive deeply inelastic scatterings (SIDIS) off a large nucleus [10–12] in terms of the suppression of final leading hadron spectra [13–16]. Phenomenological studies of experimental data on parton energy loss can yield important information on the properties of the hot or cold nuclear medium [17].

The suppression of final large transverse momentum hadron spectra in heavy-ion collisions and leading hadron spectra in SIDIS can both be described by a medium modification of the fragmentation functions (FF's) of a propagating parton. The modification is governed by a set of medium-modified QCD evolution equations that incorporate multiple parton scattering and induced gluon bremsstrahlung through medium-modified parton splitting functions [11, 18–20]. These medium-modified splitting functions lead to energy loss of the leading partons within a jet and suppression of hadrons with large fractional momenta in the final jet fragmentation functions. They also give rise to an enhancement of soft or non-leading hadrons through hadronization of the radiated gluons and partons from induced pair production. Together with the flavor conversion processes of leading partons [10, 21, 22], induced radiation and pair production can also change the hadron flavor composition of the modified jets. Inclusion of these flavor conversion processes is critical for a precise description of jet quenching effects on identified hadron spectra and extraction of medium properties in high-energy heavy-ion and electron-ion collisions.

In this Letter, we will study the effect of induced flavor con-

version on final leading hadron spectra in SIDIS off a large nucleus within the framework of high-twist approach to multiple parton scatterings and modified parton fragmentation functions. We will numerically solve a set of medium-modified Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (mDGLAP) evolution equations for parton fragmentation functions that include flavor conversion through induced gluon emission and pair production. We examine in detail how identified hadron spectra are modified. For moderate and large values of the struck quark momentum fraction  $x_B$ , the fragmenting partons in SIDIS are dominated by valence quarks ( $u$  and  $d$ ) from the target nucleus. Since constituent quarks in  $K^-$  strange mesons ( $s$ ,  $\bar{u}$ ) can only come from pair production in the shower evolution of the  $u$  and  $d$  quark, their spectra are particularly sensitive to the medium-induced flavor conversion during the propagation of the struck valence quark in the nucleus. We will show that the medium-induced flavor conversion can enhance  $K^-$  spectra with large momentum fraction to counter the effect of parton energy loss in SIDIS with moderate and large values of  $x_B$ .

Within a generalized collinear factorization [23, 24], one can calculate contributions to the differential cross section of SIDIS,  $e^-(L) + A(p) \rightarrow e^-(L') + h(p_h) + X$ , from the leading order (LO) high-twist processes in which a quark from the target nucleus is struck by the virtual photon  $\gamma^*(q)$  with momentum  $q = L - L' = (-Q^2/2q^-, q^-, \vec{0}_\perp)$  and virtuality  $Q^2$ . In the infinite momentum frame, each nucleon inside the target nucleus carries a momentum  $p = (p^+, 0, \vec{0}_\perp)$ . The struck quark with an initial momentum fraction  $x_B = Q^2/2p^+q^-$  then undergoes multiple scatterings with the remnant of the target nucleus before fragmenting into a final hadron with momentum  $p_h = (0, z_h q^-, \vec{0}_\perp)$ . The final differential cross section at LO can be expressed in terms of a collinear hard part of photon-quark scattering and a medium-modified fragmentation function (mFF)  $\tilde{D}_q^h(z_h, Q^2)$  [10].

Following the approach for evolution of the renormalized FF's in vacuum, we can also sum the leading-log and twist-four medium corrections and arrive at the mDGLAP evolution

equations for mFF's [10, 18, 25],

$$\frac{\partial \tilde{D}_q^h(z_h, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[ \tilde{\gamma}_{q \rightarrow qg}(z, Q^2) \tilde{D}_q^h\left(\frac{z_h}{z}, Q^2\right) + \tilde{\gamma}_{q \rightarrow gq}(z, Q^2) \tilde{D}_g^h\left(\frac{z_h}{z}, Q^2\right) \right], \quad (1)$$

$$\frac{\partial \tilde{D}_g^h(z_h, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[ \tilde{\gamma}_{g \rightarrow gq}(z, Q^2) \tilde{D}_g^h\left(\frac{z_h}{z}, Q^2\right) + \sum_{q=1}^{2n_f} \tilde{\gamma}_{g \rightarrow q\bar{q}}(z, Q^2) \tilde{D}_q^h\left(\frac{z_h}{z}, Q^2\right) \right], \quad (2)$$

which are similar to the DGLAP equations for vacuum FF's. The differences here from the vacuum DGLAP equations are the medium-modified splitting functions  $\tilde{\gamma}_{a \rightarrow bc}$ ,

$$\tilde{\gamma}_{a \rightarrow bc}(z, Q^2) = \gamma_{a \rightarrow bc}(z) + \Delta\gamma_{a \rightarrow bc}(z, Q^2), \quad (3)$$

which contain both the parton splittings in vacuum  $\gamma_{a \rightarrow bc}(z)$  and medium-induced ones  $\Delta\gamma_{a \rightarrow bc}(z, Q^2)$ , whose detailed expressions can be found in Refs. [18, 25]. Note that the medium-induced splitting functions  $\Delta\gamma_{a \rightarrow bc}$  depend on the jet transport parameter  $\hat{q}$  integrated over the path length of the quark propagation. For example,

$$\begin{aligned} \Delta\gamma_{q \rightarrow qg}(z, \ell_T^2) &= \frac{1}{\ell_T^2 + \mu_D^2} [C_A(1-z)(1+(1-z)^2)/z \\ &+ C_F z(1+(1-z)^2)] \\ &\times \int dy^- \hat{q}(y^-) 4 \sin^2(x_L p^+ y^- / 2), \end{aligned} \quad (4)$$

is the medium-induced quark splitting function, where  $\ell_T$  is the relative transverse momentum of the final partons and  $x_L = \ell_T^2 / 2p^+ q^- z(1-z)$  is the fractional light-cone momentum carried by the hard parton from the target nucleus that induces the parton splitting,  $y^-$  is the light-cone coordinate of the target nucleons involved in the secondary scattering and  $\mu_D$  represents beam partons' average intrinsic transverse momentum inside a nucleon. The jet transport parameter  $\hat{q}$  here arises from the twist-four quark-gluon correlation function in a factorized form as assumed in Ref. [18]. The medium-induced quark-to-gluon and gluon-to-quark splitting functions in Eqs. (1) and (2) couple the quark and gluon fragmentation functions through the mDGLAP equations. These are where the medium-induced flavor conversion occurs and will lead to a change in the flavor composition in the mFF's of the quark jets in SIDIS.

To solve the mDGLAP evolution equations in Eqs. (1) and (2), we have to provide the initial conditions of mFF's at a given initial scale  $Q_0$ . These initial conditions are not calculable in perturbative QCD (pQCD) and have to be given by a model assumption. Instead of using vacuum FF's for the initial conditions [26], we proposed a *convoluted* model [20] in order to take into account of parton energy loss for partons with virtualities below  $Q_0^2$ . The *convoluted* initial conditions are obtained from the convolution of vacuum FF's at the initial scale  $Q_0^2$  and the quenching weight due to induced gluon

radiation,

$$\begin{aligned} \tilde{D}_g^h(z, Q_0^2) &= \int_0^1 d\epsilon P_g(\epsilon, Q_0^2) \frac{1}{1-\epsilon} D_g^h\left(\frac{z}{1-\epsilon}, Q_0^2\right) \\ &+ \int_0^1 d\epsilon G^g(\epsilon, Q_0^2) \frac{1}{\epsilon} D_g^h\left(\frac{z}{\epsilon}, Q_0^2\right), \end{aligned} \quad (5)$$

$$\begin{aligned} \tilde{D}_q^h(z, Q_0^2) &= \int_0^1 d\epsilon P_q(\epsilon, Q_0^2) \frac{1}{1-\epsilon} D_q^h\left(\frac{z}{1-\epsilon}, Q_0^2\right) \\ &+ \int_0^1 d\epsilon G^q(\epsilon, Q_0^2) \frac{1}{\epsilon} D_g^h\left(\frac{z}{\epsilon}, Q_0^2\right), \end{aligned} \quad (6)$$

where the quenching weight  $P_a(\epsilon, Q_0^2)$  represents the probability of total fractional energy loss  $\epsilon$  by the initial parton  $a$  due to induced gluon radiation and  $G^a(\epsilon)$  represents the spectrum distribution of the radiated gluons with fractional energy  $\epsilon$ . The vacuum FF's  $D_a^h(z, Q^2)$  are taken from the HKN parametrization [27]. The quenching weight  $P_a(\epsilon, Q_0^2)$  is calculated from a Poisson convolution of the single gluon spectrum  $dN_g^a/dz$  at scale  $Q_0^2$ ,

$$\begin{aligned} P_a(\epsilon, Q_0^2) &= \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=1}^n \int_0^1 dz_i \frac{dN_g^a}{dz_i}(Q_0^2) \delta\left(\epsilon - \sum_{i=1}^n z_i\right) \\ &\times \exp\left[-\int_0^1 dz \frac{dN_g^a}{dz}(Q_0^2)\right], \end{aligned} \quad (7)$$

under the assumption that the number of independent gluon emissions satisfies the Poisson distribution. We use Monte Carlo simulations to calculate the quenching weight  $P_a(\epsilon, Q_0^2)$ . This method also enables us to record the energy fraction of each radiated gluon and then obtain the gluon energy spectrum  $G^a(\epsilon)$  from multiple induced emissions. With  $G^a(\epsilon)$ , we can include contributions from the fragmentation of radiated gluons to the initial conditions and also ensure the momentum conservation at the same time. Using such initial conditions for the mDGLAP equations, we can describe the HERMES data [15] better as compared to other models for initial conditions. Details can be found in Ref. [20].

Jet quenching in SIDIS is measured experimentally via the suppression of leading hadron spectra. The nuclear modification factor  $R_A^h$  for hadron spectra is defined in terms of a ratio of hadron yields per DIS event  $N^h/N^e$  for a nuclear target  $A$  to that for a deuterium target  $D$  [13–16],

$$R_A^h(\nu, Q^2, z) = \left[ \frac{N^h(\nu, Q^2, z)}{N^e(\nu, Q^2)} \right]_A / \left[ \frac{N^h(\nu, Q^2, z)}{N^e(\nu, Q^2)} \right]_D. \quad (8)$$

Hadron yields per DIS event  $N^h/N^e$  from LO pQCD can be related to the nuclear modified FF's  $\tilde{D}_q^h(z, Q^2)$  from the mDGLAP evolution equations in Eqs. (1) and (2),

$$\left. \frac{N^h(\nu, Q^2, z)}{N^e(\nu, Q^2)} \right|_A = \frac{\sum e_q^2 q(x_B, Q^2) \tilde{D}_q^h(z, Q^2)}{\sum e_q^2 q(x_B, Q^2)} \Big|_A, \quad (9)$$

where  $q(x, Q^2)$  is the quark distribution function inside the nucleus and  $e_q$  is the quark's charge. The mFF's  $\tilde{D}_q^h(z, Q^2)$  are obtained from the numerical solution of the mDGLAP

equations for a given propagation path or interaction point of the photon-quark scattering inside the nucleus which is then averaged over the nucleus' volume,

$$\tilde{D}_q^h(z, Q^2) = \frac{1}{A} \int d^2b dy_0 \tilde{D}_q^h(z, y_0, b, Q^2) \rho_A(y_0, b), \quad (10)$$

where  $\rho_A(y, b)$  is the Woods-Saxon nuclear density distribution which is normalized as  $\int dy d^2b \rho_A(y, b) = A$ . The jet transport parameter  $\hat{q}$  along the propagation path of the struck quark is assumed to be proportional to the nuclear density,  $\hat{q}(y, b) = \hat{q}_0 \rho_A(y, b) / \rho_A(0, 0)$ , where  $\hat{q}_0$  is the value of  $\hat{q}$  at the center of the nucleus. With the *convoluted* initial conditions, we were able to describe the HERMES data on the hadron suppression [15] well and obtained  $\hat{q}_0 = 0.020 \pm 0.005$  GeV<sup>2</sup>/fm [18–20] from a  $\chi^2$  fit. We will use the central value of  $\hat{q}_0$  from this fit for all numerical calculations in this study.

In search for a clear signal of the medium-induced flavor conversion in the final hadron spectra, we choose the charged strange meson  $K^-$  since its constituent quarks ( $s$  and  $\bar{u}$ ) are not the same as any of the struck valence quarks ( $u$  and  $d$ ) of the target nucleus. In Fig. 1, we show the vacuum FF's of different partons to  $K^-$  as a function of the momentum fraction  $z$ . The vacuum FF's of the non-constituent quarks fall off exponentially at large  $z$  and are much smaller than that of gluon and constituent quarks which are relatively flat.

Because of the exponential fall-off of its vacuum FF,  $K^-$  production from an initial non-constituent quark should be significantly suppressed at large  $z$  due to parton energy loss. On the other hand, contributions to the final  $K^-$  spectrum from gluons and constituent quarks via medium-induced flavor conversion become significant. They offset the suppression of  $K^-$  spectrum due to energy loss of an initial non-constituent quark and can even lead to an enhancement of the final effective mFF for  $K^-$  at large  $z$  as shown in Fig. 2, where we plot ratios of mFF's to the vacuum ones for  $K^-$  from different initial partons. Parton energy loss also leads to suppression of leading  $K^-$  production from an initial gluon or constituent quark jet. However, their contributions to the  $K^-$  spectrum are still much larger than that from medium-induced gluon and quark pairs due to the flat behavior of their vacuum FF's. The final effective mFF's for  $K^-$  from an initial gluon or constituent quark jet are therefore suppressed at large  $z$  as seen in Fig. 2. At intermediate values of  $z$  where the vacuum FF's are about the same for all partons, the mFF's are all suppressed due to parton energy loss. At small  $z$ , mFF's are all enhanced by the medium-induced gluon emission and quark-pair production.

To investigate the sensitivity of charged kaon spectra to the medium-induced flavor conversion in SIDIS off a nucleus, we focus on SIDIS off a Pb nucleus at moderate and large  $x_B$  so that the struck quarks are mostly valence quarks ( $u$  and  $d$ ) from the nucleus. In this region of  $x_B$ , the quark distribution in a nucleon is indeed dominated by valence quarks as shown in Table I. One should also in principle consider nuclear modification of quark distributions inside a nucleus. The EMC effect [28], however, is mostly independent of the quark flavor. Its effect on hadron yields per DIS event in Eq. (9) will mostly cancel. We will simply use the vacuum parton distri-

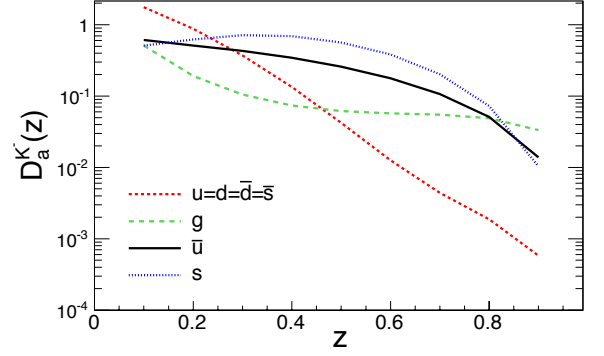


FIG. 1: (color online) Parton fragmentation functions to  $K^-$  in vacuum at  $Q^2 \approx 10$  GeV<sup>2</sup> from the HKN parametrization [27].

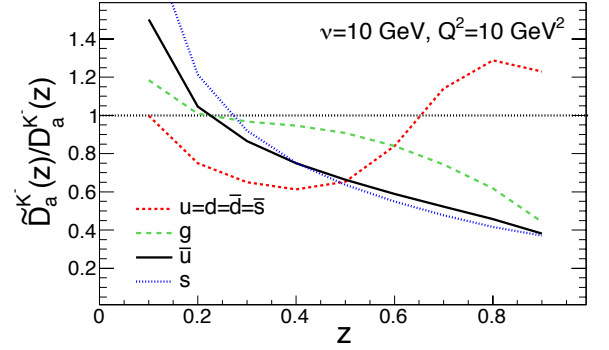


FIG. 2: (color online) Ratios of mFF's to the vacuum ones for  $K^-$  from different partons with initial energy  $\nu = 10$  GeV and  $Q^2 = 10$  GeV<sup>2</sup> in SIDIS off Pb.

bution functions from the CTEQ6 parameterization [29] and consider only the isospin difference in a nucleus which has negligible effects on the final kaon spectra.

$q(x_B, Q^2)$	$\bar{s}, s$	$\bar{d}$	$\bar{u}$	$u$	$d$
$x_B = 0.5$	0.0018	0.0029	0.0053	0.5331	0.1532
$x_B = 0.1$	0.4790	1.3961	0.9262	5.6736	3.7867

TABLE I: Values of quark distributions inside a proton from the CTEQ6 parameterization [29] at  $Q^2=10$  and 2 GeV<sup>2</sup> for  $x_B = 0.5$  and  $x_B = 0.1$ , respectively.

In Fig. 3, we show the nuclear modification factor for charged kaons as a function of momentum fraction  $z$  for different values of the initial quark energy  $q^- = \nu$  at  $x_B = 0.1$ . These kinematics are similar to that in the HERMES experiment [15]. The spectra of both kaons are suppressed due to parton energy loss in the nuclear medium. The suppression decreases and eventually gives away to an enhancement at small  $z$  due to the softening of the mFF's from radiated gluons and induced pair production. However, the suppression of  $K^-$  at large  $z$  is flatter which is distinctly different from that of  $K^+$ . This difference is already present in the HERMES data [15, 20] and is a strong indication of the onset of medium-induced flavor conversion. The effect is, however,

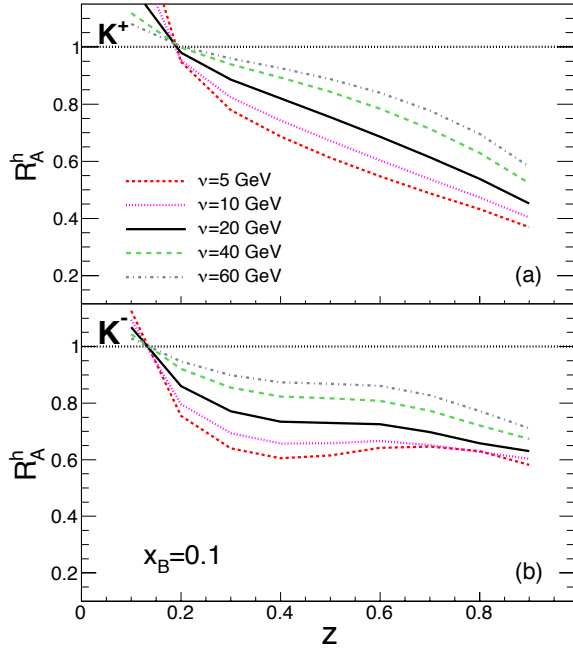


FIG. 3: (color online) The nuclear modification factor for (a)  $K^+$  and (b)  $K^-$  for different initial quark energy  $\nu$  in SIDIS at  $x_B = 0.1$ .

not very significant at  $x_B \leq 0.1$  because there are still finite fractions of initial quarks with the constituent flavor of  $K^-$  (see Table I) whose fragmentation dominates the  $K^-$  spectrum at large  $z$  in spite of their energy loss. Contributions from the induced flavor conversion in non-constituent initial quarks only offset partially the effect of energy loss of the initial constituent quarks and lead to a suppression factor for  $K^-$  that is flatter than  $K^+$ .

In DIS events with larger values of  $x_B$ , the initial struck quarks are completely dominated by valence quarks of the nucleus. One should expect to see enhancement of  $K^-$  due to medium-induced flavor conversion in the mFF's as shown in Fig. 2. This is indeed confirmed in Fig. 4 where we show nuclear modification factors for kaons at  $x_B = 0.5$ . The suppression of  $K^+$  is about the same as that at  $x_B = 0.1$  due to the weak dependence of medium modification on  $Q^2$  [19, 20]. The nuclear modification factor for  $K^-$  is, however, completely different. The  $K^-$  spectrum is suppressed at intermediate values of  $z$  due to parton energy loss. But at large  $z$ , the modification factor starts to increase and approaches or exceeds 1, due to contributions from gluons and constituent quarks via medium-induced flavor conversion. If we turn off the flavor conversion processes ( $q \rightarrow gq$  and  $g \rightarrow q\bar{q}$ ) in the mDGLAP equations, the rise of the nuclear modification factor for  $K^-$  at large  $z$  disappears. Since both parton energy loss and flavor conversion are proportional to jet-medium interaction, the behavior of the modification factor for  $K^-$  at intermediate and large  $z$  is sensitive to the jet transport parameter  $\hat{q}$ . Experimental study of this phenomenon therefore provides another independent constraint on the jet transport parameter in the nuclear medium. The study can also be applied to  $\bar{K}^0$  spectra. However, the effect in  $\bar{K}_S^0$  spectra will be reduced because it is a mixture of  $K^0$  and  $\bar{K}^0$ .

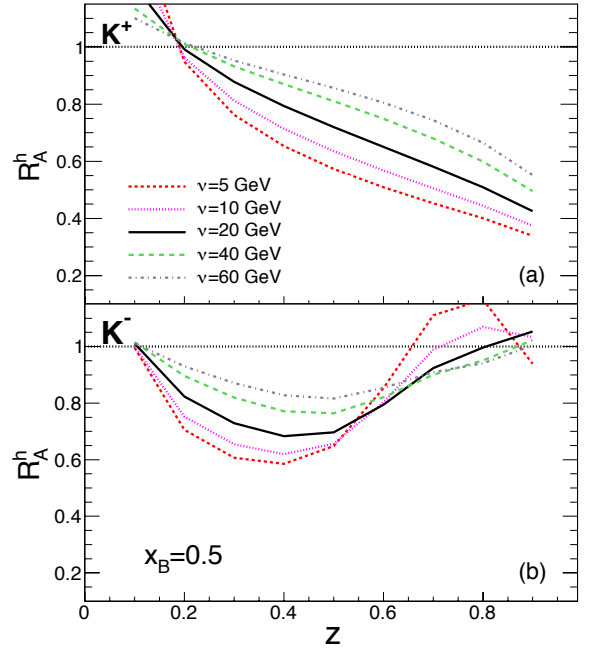


FIG. 4: (color online) The same as Fig. 3 except for  $x_B = 0.5$ .

In conclusion, we have carried out a numerical study of the mDGLAP evolution of mFF's within a high-twist approach to parton propagation in medium. We discover that the nuclear modification factor for  $K^-$  in SIDIS at large  $x_B$  is very sensitive to the medium-induced flavor conversion. Instead of increased suppression like other hadrons (pions and  $K^+$ ) due to parton energy loss of the leading quarks, the nuclear modification factor for  $K^-$  is shown to rise and can exceed 1 at large values of  $z$  due to the proliferation of constituent quarks ( $s$  and  $\bar{u}$ ) and gluons from the induced flavor conversion to counter the effect of parton energy loss. This novel behavior is also found to be sensitive to the value of the jet transport parameter. Therefore, experimental measurements of such a phenomenon in a future electron-ion collider (EIC) can provide another independent probe of the properties of nuclear medium at high energies. It can also help us to understand the flavor hierarchy of jet quenching phenomena in high-energy heavy-ion collisions [30–32]. A major numerical uncertainty in our quantitative estimate of the effect of medium-induced flavor conversion on the final  $K^-$  spectrum at large  $z$  is the parameterization of vacuum FF's for kaons. Such uncertainty can also be reduced by future experimental measurements at EIC in the large  $x_B$  regions.

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